A new scheme for buckling analysis of bidirectional functionally graded Euler beam having arbitrary thickness variation rested on Hetenyi elastic foundation

Abbas Heydari, Abdolrahim Jalali*

Mechanical Engineering Department, Shahid Rajaee Teacher Training University, Tehran, Iran.
P.O.B. 5517910179 Tehran, Iran, maligoodarz@srttu.edu

ARTICLE INFORMATION

Original Research Paper
Received 01 October 2016
Accepted 28 November 2016
Available Online 04 January 2017

Keywords:
Buckling analysis
Tapered BFG beam
Spectral methods
Calculus of variations
Hetenyi elastic foundation

ABSTRACT

In current work, for the first time buckling analysis of bidirectional functionally graded (BFG) Euler beam having arbitrary thickness variation rested on Hetenyi elastic foundation is presented. Moreover, a new scheme based on calculus of variations and collocation method for converting the buckling problem to an algebraic system of equations is proposed. The mentioned scheme leads to obtaining the buckling characteristic equation of beam and therefore the first buckling loads are obtained. Various conditions including variation of mechanical properties across the thickness and through the axis, arbitrary thickness variation, Hetenyi elastic foundation, special boundary conditions like the shear hinge and clamped boundary conditions like the clamped, simply supported, clamped-simply supported and cantilever beams are considered to show the compatibility of proposed scheme with the various circumstances. The fast convergence and compatibility with the various circumstances are the advantages of the proposed technique. Due to lack of similar studies in the literature, the same exercises are conducted by using the Spectral Ritz method for pursuing the validity of the proposed scheme. The same basis is used for Spectral Ritz and proposed methods. Excellent agreement is found between the results of well-known Spectral Ritz method and the results of proposed scheme, which validates the outcome of the proposed technique.

Please cite this article using:
شکل ۱: ویژگی‌های هندسی منجر

$$u_b = \frac{B_0}{2} \int_0^L \left( \frac{e^{\frac{x^2}{2}}}{w^2} \right) \left( 1 + \beta \frac{x}{L} \right)^2 \left( \phi_1 \left( 1 + \beta \frac{x}{L} \right)^2 + \phi_2 \left( 1 + \beta \frac{x}{L} \right)^2 + \phi_3 \phi_3 \right) dx$$ (6)

$$E = \frac{1}{2} \int_0^L \frac{y}{w}^2 \left( \left( E_c - E_m \right) \left( \left( \frac{n + 2}{n + 3} \right)^2 + \left( \frac{n - 1}{n + 2} \right)^2 \right) + \left( \frac{n}{n + 3} \right)^2 \left( E_m + E_m \right) \right) du$$ (3)

$$h = h_0 \left( 1 + \beta \frac{x}{L} \right)^2$$ (4)

$$u_b = \frac{F_0}{2} \int_0^L \left( \frac{e^{\frac{x^2}{2}}}{w^2} \right) \left( 1 + \beta \frac{x}{L} \right)^2 \left( \phi_1 \left( 1 + \beta \frac{x}{L} \right)^2 + \phi_2 \left( 1 + \beta \frac{x}{L} \right)^2 + \phi_3 \phi_3 \right) dx$$ (5)

$$E = \frac{1}{2} \int_0^L \frac{y}{w}^2 \left( \left( E_c - E_m \right) \left( \left( \frac{n + 2}{n + 3} \right)^2 + \left( \frac{n - 1}{n + 2} \right)^2 \right) + \left( \frac{n}{n + 3} \right)^2 \left( E_m + E_m \right) \right) du$$ (3)

$$h = h_0 \left( 1 + \beta \frac{x}{L} \right)^2$$ (4)
خواص مکانیکی یکسان، یافته و نحوه ندارد [17] همچنین، در تحلیل
کامپیوتری صحت با دقت مناسب منجر به اجرای
فشار ویرایشگر و تغییر شکل بسته کامپیوتری می‌شود، نتایج گردش
است [18] از طریق الگوریتم بسته کامپیوتری به فرم رابطه (11) می‌باشد، باید بر
ستایش تغییر تا وابسته به فرمول
\[ EI = \frac{1}{2} \int (kw + Elw'(4)) \, dw \]
کار ثابت گرایی سری در تغییر دقت است. با
استفاده از دو جدول بسط تابع، ارزیاره هر فهرست
\( \Omega \) با منفی یک ثابت خارجی، به صورت معمول (12) می‌شود.
\[ \Omega = F \int_0^1 (1 - w(1))^2 \, dx = - F \frac{2}{10} \int_0^1 (w(1))^2 \, dx \]
ارزیابی پژوهشی کل برای یک مجموعه از بررسی‌های دختره شده و بر رفتار
است.
\[ I = \frac{1}{2} \int_0^1 \left( \phi_x \frac{\partial \phi}{\partial x} (w''(2))^2 \right) (1 + \frac{\beta (X)}{\beta})^3 + \]
\[ (kw + Elw'(4)) \, dw = F \frac{2}{10} \int_0^1 (w(1))^2 \, dx \]
یا به صورت معمول (14) بر حسب مقدار نهایی استفاده شده در میانه
\( \phi = A_0 \Phi_1 (1 - \frac{x}{2}) \)
به دریافت یک نتیجه پیشنهادی که در صفحه هدفمندی کل برای شیپ و
می‌تواند با استفاده از دو جدول نهایی دیده شده در
\[ \phi = \phi_1 \Phi_1 + \phi_2 \Phi_2 + \phi_3 \Phi_3 \]
یا به صورت معمول (14) پیشنهاد شده.
\[ \phi = A_0 \Phi_1 (1 - \frac{x}{2}) \]
به دریافت یک نتیجه پیشنهادی که در صفحه هدفمندی کل برای شیپ و
می‌تواند با استفاده از دو جدول نهایی دیده شده در
\[ \phi = \phi_1 \Phi_1 + \phi_2 \Phi_2 + \phi_3 \Phi_3 \]
یا به صورت معمول (14) پیشنهاد شده.
\[ \phi = A_0 \Phi_1 (1 - \frac{x}{2}) \]
به دریافت یک نتیجه پیشنهادی که در صفحه هدفمندی کل برای شیپ و
می‌تواند با استفاده از دو جدول نهایی دیده شده در
\[ \phi = \phi_1 \Phi_1 + \phi_2 \Phi_2 + \phi_3 \Phi_3 \]
یا به صورت معمول (14) پیشنهاد شده.
\[ \phi = A_0 \Phi_1 (1 - \frac{x}{2}) \]
به دریافت یک نتیجه پیشنهادی که در صفحه هدفمندی کل برای شیپ و
می‌تواند با استفاده از دو جدول نهایی دیده شده در
\[ \phi = \phi_1 \Phi_1 + \phi_2 \Phi_2 + \phi_3 \Phi_3 \]
یا به صورت معمول (14) پیشنهاد شده.
\[ \phi = A_0 \Phi_1 (1 - \frac{x}{2}) \]
به دریافت یک نتیجه پیشنهادی که در صفحه هدفمندی کل برای شیپ و
می‌تواند با استفاده از دو جدول نهایی دیده شده در
\[ \phi = \phi_1 \Phi_1 + \phi_2 \Phi_2 + \phi_3 \Phi_3 \]
یا به صورت معمول (14) پیشنهاد شده.
6. The roots of characteristic polynomials

The roots of characteristic polynomials are significant in understanding the behavior of linear systems. The roots of a characteristic polynomial provide information about the stability and transient response of the system.

For a linear system described by the differential equation:

\[ a_0 y''(t) + a_1 y'(t) + a_2 y(t) = 0 \]

the characteristic polynomial is:

\[ a_0 s^2 + a_1 s + a_2 = 0 \]

The roots of this polynomial, \( s_1 \) and \( s_2 \), give the natural frequencies of the system.

\[ s_1, s_2 = \frac{-a_1 \pm \sqrt{a_1^2 - 4a_0a_2}}{2a_0} \]

These roots can be real or complex, indicating whether the system is stable, marginally stable, or unstable, respectively.

\[ \phi = \frac{1}{\sqrt{1 - k^2}} \]

where \( k \) is the system's damping ratio.

The roots of the characteristic polynomial can also be used to analyze the transient response of a system, as they determine the form of the exponential functions that make up the solution.

\[ y(t) = e^{s_1t} + e^{s_2t} \]

For a second-order system, the roots determine whether the system oscillates (complex roots), damps out (real roots), or grows without bound (negative real roots).

\[ \phi = \frac{1}{\sqrt{1 - k^2}} \]

In conclusion, the roots of the characteristic polynomial are crucial for analyzing the dynamic behavior of linear systems.

\[ \phi = \frac{1}{\sqrt{1 - k^2}} \]

where \( k \) is the system's damping ratio.

The roots of the characteristic polynomial can also be used to analyze the transient response of a system, as they determine the form of the exponential functions that make up the solution.

\[ y(t) = e^{s_1t} + e^{s_2t} \]

For a second-order system, the roots determine whether the system oscillates (complex roots), damps out (real roots), or grows without bound (negative real roots).

\[ \phi = \frac{1}{\sqrt{1 - k^2}} \]

In conclusion, the roots of the characteristic polynomial are crucial for analyzing the dynamic behavior of linear systems.

\[ \phi = \frac{1}{\sqrt{1 - k^2}} \]

where \( k \) is the system's damping ratio.

The roots of the characteristic polynomial can also be used to analyze the transient response of a system, as they determine the form of the exponential functions that make up the solution.

\[ y(t) = e^{s_1t} + e^{s_2t} \]

For a second-order system, the roots determine whether the system oscillates (complex roots), damps out (real roots), or grows without bound (negative real roots).

\[ \phi = \frac{1}{\sqrt{1 - k^2}} \]

In conclusion, the roots of the characteristic polynomial are crucial for analyzing the dynamic behavior of linear systems.
شکل 4 تأثیر نسبت ماده در استفاده محور و ضریب ارتعاش بر الگوی کامپلیکس

شکل 5 تأثیر ضریب خصوصی تیر از مدل هرینی بر بال کامپلیکس

شکل 6 تأثیر نسبت ماده و نسبت مدول ارتعاش سرامیک بر فاصله دوجینه لوله بر ضریب دفعه واقع بر بستر الگوی کامپلیکس
The dimensionless critical buckling loads for various amounts of $n$ and $\gamma$

<table>
<thead>
<tr>
<th>$F_n L^2$</th>
<th>$\gamma$</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.41019</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>9.08665</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>9.86834</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>7.21536</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>7.71020</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>8.25268</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>6.98535</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>7.44479</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>7.97061</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>8.89609</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>7.33735</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>8.75134</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>6.83983</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>7.27231</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>7.76484</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

The dimensionless buckling loads corresponding to the first mode for various boundary conditions

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.474046</td>
<td>39.42513</td>
</tr>
<tr>
<td>9.838913</td>
<td>2.449701</td>
</tr>
</tbody>
</table>

The dimensionless buckling loads for various amounts of $k$ and $E_1$

<table>
<thead>
<tr>
<th>$F_n L^2$</th>
<th>$E_1$</th>
<th>$E_m$</th>
<th>k</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.8126</td>
<td>0.0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2.8780</td>
<td>0.5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3.9206</td>
<td>1.0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1.9024</td>
<td>0.0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2.9567</td>
<td>0.5</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>4.0073</td>
<td>1.0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1.9472</td>
<td>0.0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3.0095</td>
<td>0.5</td>
<td>15</td>
<td>0</td>
</tr>
<tr>
<td>4.0506</td>
<td>1.0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1.9920</td>
<td>0.0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2.9033</td>
<td>0.5</td>
<td>20</td>
<td>0</td>
</tr>
<tr>
<td>4.0939</td>
<td>1.0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2.1074</td>
<td>0.0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3.0970</td>
<td>0.5</td>
<td>25</td>
<td>0</td>
</tr>
<tr>
<td>4.1372</td>
<td>1.0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The dimensionless buckling loads for various amounts of $\beta$ and $\alpha$

<table>
<thead>
<tr>
<th>$F_n L^2$</th>
<th>$\beta$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>11.6849</td>
<td>0.3</td>
<td>0.4</td>
</tr>
<tr>
<td>7.45603</td>
<td>0.0</td>
<td>0.4</td>
</tr>
<tr>
<td>5.42100</td>
<td>-0.3</td>
<td>0.6</td>
</tr>
<tr>
<td>11.0008</td>
<td>0.3</td>
<td>0.2</td>
</tr>
<tr>
<td>7.13375</td>
<td>0.0</td>
<td>0.2</td>
</tr>
<tr>
<td>5.32170</td>
<td>-0.3</td>
<td>0.6</td>
</tr>
<tr>
<td>10.3674</td>
<td>0.0</td>
<td>0.2</td>
</tr>
<tr>
<td>6.84441</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>5.23364</td>
<td>-0.3</td>
<td>0.6</td>
</tr>
<tr>
<td>9.78317</td>
<td>0.3</td>
<td>0.2</td>
</tr>
<tr>
<td>6.58544</td>
<td>0.0</td>
<td>-0.2</td>
</tr>
<tr>
<td>5.15546</td>
<td>0.3</td>
<td>-0.4</td>
</tr>
<tr>
<td>9.24663</td>
<td>0.3</td>
<td>-0.4</td>
</tr>
<tr>
<td>6.35429</td>
<td>0.0</td>
<td>-0.4</td>
</tr>
<tr>
<td>5.08599</td>
<td>0.3</td>
<td>-0.4</td>
</tr>
</tbody>
</table>

The dimensionless buckling loads for various amounts of $\gamma$ and $E_1$

<table>
<thead>
<tr>
<th>$F_n L^2$</th>
<th>$\gamma$</th>
<th>$E_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.445315</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0.6996709</td>
<td>2</td>
<td>S-S</td>
</tr>
<tr>
<td>0.789138</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>0.345492</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0.559702</td>
<td>2</td>
<td>S-C</td>
</tr>
<tr>
<td>0.667076</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>0.447232</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0.437645</td>
<td>2</td>
<td>C-C</td>
</tr>
<tr>
<td>0.635137</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>1.000000</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0.605882</td>
<td>2</td>
<td>C-F</td>
</tr>
<tr>
<td>0.330255</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>1.000000</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0.421922</td>
<td>2</td>
<td>C-SHH</td>
</tr>
</tbody>
</table>


